Computing the wavespeed of soliton-like solutions in SmC* liquid crystals

L.J. Seddon and I. W. Stewart

Department of Mathematics
University of Strathclyde
Livingstone Tower
26 Richmond Street
Glasgow G1 1XH
United Kingdom

We present a novel method for numerically computing the wavespeed of a soliton-like travelling wave in SmC* liquid crystals that satisfies a parabolic partial differential equation (PDE) with a general nonlinear term (1). We show, by transforming the PDE to a co-moving frame and recasting the resulting problem in phase-space, that the original PDE can be expressed as an integral equation known as an exceptional nonlinear Volterra-type equation of the second kind. This technique is motivated by, but distinct in nature from, iterative integral methods introduced used by Chernyak (2). We choose boundary conditions by considering fixed points of the nonlinear term. By applying a simple trapezoidal method to the integral equation we generate a system of nonlinear simultaneous equations which we solve for our phase plane variable at equally spaced intervals using Newton iterates. The equally spaced phase variable solutions are then used to compute the wavespeed of the associated travelling wave. We demonstrate an algorithm for performing the necessary calculations by considering an example from liquid crystal theory, where a parabolic PDE with a nonlinear reaction term has a solution and wavespeed which are known exactly (3,4). This is compared with the numerically computed wavespeed using our new scheme and the results are compared.

References

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